The see-saw mechanism and heavy Majorana neutrino masses in an SO(10) model

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Abstract

We apply the see-saw mechanism and a SO(10) model to neutrino masses and mixing in order to estimate the heavy Majorana masses. We discuss shortly the decay modes of heavy Majorana neutrinos and calculate their contribution to the lepton number violating processes $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$.

1 Introduction

The Standard Model fermionic fields do not include the right-handed neutrino field and so it is impossible to have a Dirac mass term for this particle provided it is not introduced *ad hoc*. In this case we will have the following lagrangian mass term: $\mathcal{L} = f_\nu \bar{l}_L \nu_R \phi + h.c. \to m_D \overline{\nu_L} \nu_R + h.c.$, where $m_D = \frac{1}{\sqrt{2}} v f_\nu$ is the Dirac mass. To have a neutrino mass of order of eV we should have a Yukawa coupling $f_\nu \sim 10^{-11}$ which seems very unlikely.

The best explanation of the lightness of neutrino comes from the see-saw mechanism [4].This mechanism works in a natural way in the framework of grand unification theories such as $SO(10)$ but also in the Standard Model, if we include a right-handed neutrino field. The mechanism is based on three simple hypotheses, motivated in the grandunification model: 1) a zero Majorana mass for left-handed neutrinos ($m_L \simeq 0$), 2) the neutrino Dirac mass should be comparable to the charged fermion masses ($m_D \simeq m^f$) and 3) the Majorana mass of ν_R should be much larger than the Dirac ones ($m_R \simeq \mathcal{M} \gg m_D$). In this way the lagrangian mass terms, in the one generation case, will be:

$$\mathcal{L}^{D+M} = -m_D \overline{\nu_R} \nu_L - \frac{1}{2} m_R \overline{\nu_R} \nu_R^c + h.c. = -m^f \overline{\nu_R} \nu_L - \frac{1}{2} \mathcal{M} \overline{\nu_R} \nu_R^c + h.c. = -\frac{1}{2} \overline{n_L^c} M n_L + h.c. \quad (1)$$

with:

$$M \equiv \begin{pmatrix} 0 & m^f \\ m^f & \mathcal{M} \end{pmatrix} \quad (2)$$

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The eigenvalues of this mass matrix will be the neutrino masses:

$$m_1 \simeq -\frac{(m^f)^2}{\mathcal{M}} \ll m^f \qquad m_2 \simeq \mathcal{M} \tag{3}$$

obtaining a very low mass, which would explain the lightness of neutrino, and a very high mass, for a superheavy neutrino.

In the three generations case we will have a 6×6 mass matrix:

$$M = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \tag{4}$$

where M_D and M_R are 3×3 matrixes (M_R is also symmetric). We may block diagonalize M by a unitary trasformation [5], obtaining the 3×3 mass matrixes for light and heavy neutrinos [6, 2]:

$$M_{light} \simeq -M_D^T M_R^{-1} M_D \tag{5}$$

$$M_{heavy} \simeq M_R \tag{6}$$

Thus heavy neutrino masses are the eigenvalues of the right-handed Majorana mass matrix.

2 Mixing of quarks and leptons in SO(10)

The mixing of fermions derives from the fact that the mass matrix is not diagonal and so the weak eigenstates are different from the mass eigenstates. In the quark sector the mass matrixes of up quarks M_u and of down quarks M_d may be diagonalized by a biunitary transformation:

$$\begin{aligned} L_u^\dagger M_u R_u &= M_u^{diag} \\ L_d^\dagger M_d R_d &= M_d^{diag} \end{aligned} \tag{7}$$

and thus the Cabibbo-Kobayashi-Maskawa matrix is:

$$K = L_u^\dagger L_d \tag{8}$$

and the mixing of quarks is:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = K \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{9}$$

where the weak eigenstates are labelled by $'$. In the same way, in the leptonic sector we have:

$$\begin{aligned} L_\nu^\dagger M_\nu R_\nu &= M_\nu^{diag} \\ L_l^\dagger M_l R_l &= M_l^{diag} \end{aligned} \tag{10}$$

where M_ν is the mass matrix of light neutrinos and M_l is the mass matrix of charged leptons; the mixing matrix of neutrinos is:

$$U^\dagger = L_\nu^\dagger L_l \tag{11}$$

and thus we obtain:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (12)$$

The problem is that it is the 6×6 mass matrix (4) that should be diagonalized not the mass matrix of light neutrinos. Yet it can be shown [8] that the same result is achieved if we consider the 3×3 mass matrix of light neutrinos.

The 16 left-handed fields of one generation of fermions transforms like a 16-component SO(10) spinor. The product $16 \otimes 16 = 10 \oplus 120 \oplus 126$ leaves open the possibility for a 10-, a 120- or a 126-Higgs multiplet. The only SU(5) singlet is contained in 126 and leads to a very large Majorana mass for ν_R. Therefore we exclude the possibility of SU(2) doublet Higgs contained in 126 and limit our choice, for simplicity, to the Higgs mesons of representation 10.

In the SO(10) model [9, 10, 11], using a Higgs decuplet, we obtain symmetric mass matrix and $M_d = M_l$ and $M_u = M_D$, where M_d and M_u are the mass matrix of *down* and *up* quarks, M_l the mass matrix of charged leptons and M_D the Dirac mass matrix of neutrino [12, 13, 14]. Thus the mass matrix of light neutrino will be $M_\nu = M_u M_R^{-1} M_u$. As a consequence we obtain $L_l = L_d$ and for the matrix L_ν we can write:

$$\begin{aligned} M_\nu &= M_u M_R^{-1} M_u \\ &= L_u M_u^{diag} R_u^\dagger M_R^{-1} L_u M_u^{diag} R_u^\dagger \end{aligned} \qquad (13)$$

Defining $M_1 = M_u^{diag} R_u^\dagger M_R^{-1} L_u M_u^{diag}$ and calling A and A_1 the matrixes which diagonalize it, we obtain:

$$\begin{aligned} M_\nu &= L_u M_1 R_u^\dagger \\ &= L_u A M_\nu^{diag} A_1^\dagger R_u^\dagger && (14) \\ M_\nu &= L_\nu M_\nu^{diag} R_\nu^\dagger && (15) \end{aligned}$$

Thus we may assume:

$$L_\nu = L_u A \qquad L_\nu^\dagger = A^\dagger L_u^\dagger \qquad (16)$$

and so:

$$\boxed{U^\dagger = A^\dagger L_u^\dagger L_l = A^\dagger K} \qquad (17)$$

We think the importance of Majorana mass matrix of right-handed neutrinos is worth being stressed and consequently of the heavy Majorana neutrinos. As a matter of fact, in their absence the see-saw mechanism cannot be used and besides having equality between neutrino and quark masses, we obtain equality between their mixing, which is experimentally unacceptable.

Moreover, equation (17) can show that the quadratic and linear see-saw models [7] are not correct [8]. In the quadratic see-saw the Majorana mass matrix of right-handed neutrinos is taken as proportional to the identity matrix and all the heavy neutrinos have the same mass. In this case we can see that the matrix A is equal to the identity matrix and therefore, even if we can use the see-saw mechanism, we obtain that the mixing of quarks and neutrinos is

the same. Thus we can say that not all the heavy neutrinos have the same mass and that the quadratic see-saw is not correct. Likewise it is possible to see that also the linear see-saw is not correct, so the Majorana mass matrix of right-handed neutrinos is different from the mass matrix of up quarks.

3 Masses of heavy Majorana neutrinos

In this section we are going to try to find a form for the mass matrix of the heavy Majorana neutrinos (M_R) and consequently an estimate of their masses.

From eq. (6),(13) and (14) we obtain (assuming M_u and M_R real matrixes and remembering they are simmetric):

$$M_{heavy} \simeq M_R = L_u M_u^{diag} A M_\nu^{diag-1} A^\dagger M_u^{diag} R_u^\dagger \tag{18}$$

The problem is that we do not know the form of L_u, R_u and A. But if we assume $K \simeq I$ and $L_u \simeq R_u \simeq I$ [15] we have $A \simeq U$:

$$M_{heavy} = M_u^{diag} U M_\nu^{diag-1} U^\dagger M_u^{diag} \tag{19}$$

The form of neutrino mixing matrix depends on the kind of solution of solar neutrinos. In this work we consider LMA-solution, VO-solution and SMA-solution. For the LMA-solution ($\Delta m_{12}^2 \simeq 3 \cdot 10^{-5}\ eV^2$) and the VO-solution ($\Delta m_{12}^2 \simeq 4 \cdot 10^{-10}\ eV^2$) we assume the following neutrino mixing matrix form:

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{20}$$

$$M_{heavy} = \begin{pmatrix} \frac{m_u^2}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right) & \frac{m_u m_c}{2\sqrt{2}}\left(\frac{1}{m_1}-\frac{1}{m_2}\right) & \frac{m_u m_t}{2\sqrt{2}}\left(\frac{1}{m_2}-\frac{1}{m_1}\right) \\ \frac{m_u m_c}{2\sqrt{2}}\left(\frac{1}{m_1}-\frac{1}{m_2}\right) & \frac{m_c^2}{4}\left(\frac{1}{m_1}+\frac{1}{m_2}+\frac{2}{m_3}\right) & \frac{m_c m_t}{4}\left(\frac{2}{m_3}-\frac{1}{m_1}-\frac{1}{m_2}\right) \\ \frac{m_u m_t}{2\sqrt{2}}\left(\frac{1}{m_2}-\frac{1}{m_1}\right) & \frac{m_c m_t}{4}\left(\frac{2}{m_3}-\frac{1}{m_1}-\frac{1}{m_2}\right) & \frac{m_t^2}{4}\left(\frac{1}{m_1}+\frac{1}{m_2}+\frac{2}{m_3}\right) \end{pmatrix} \tag{21}$$

For the SMA-solution ($\Delta m_{12}^2 \simeq 5 \cdot 10^{-6}\ eV^2$) we assume, instead:

$$U \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{22}$$

As a consequence, resorting to (19), we have:

$$M_{heavy} = \begin{pmatrix} \frac{m_u^2}{m_1} & 0 & 0 \\ 0 & \frac{m_c^2}{2}\left(\frac{1}{m_2}+\frac{1}{m_3}\right) & \frac{m_c m_t}{2}\left(\frac{1}{m_3}-\frac{1}{m_2}\right) \\ 0 & \frac{m_c m_t}{2}\left(\frac{1}{m_3}-\frac{1}{m_2}\right) & \frac{m_t^2}{2}\left(\frac{1}{m_2}+\frac{1}{m_3}\right) \end{pmatrix} \tag{23}$$

The eigenvalues of these matrixes are the masses of heavy neutrinos. There is a problem: we only know Δm_{ij}^2 from the analysis of solar and atmospheric neutrinos ($\Delta m_{13}^2 \simeq 3 \cdot 10^{-3}\ eV^2$); thus we have to consider the four possible relationships between the light neutrino masses m_i:

1. $m_1 \ll m_2 \ll m_3$
2. $m_1 \simeq m_2 \ll m_3$
3. $m_1 \simeq m_2 \gg m_3$
4. $m_1 \simeq m_2 \simeq m_3 \gg \Delta m_{13}^2$

By numerical calculation we obtain the results reported in table 1.

Table 1: Estimate of heavy neutrinos masses (all masses are in GeV)

Solutions	Relations among m_i	M_1	M_2	M_3
LMA	$m_1 \ll m_2 \ll m_3$	$3 \cdot 10^6$	$6 \cdot 10^{10}$	$\gg 10^{15}$
	$m_1 \simeq m_2 \ll m_3$	$2 \cdot 10^5 - 2 \cdot 10^6$	$4 \cdot 10^{10}$	$2 \cdot 10^{14} - 2 \cdot 10^{15}$
	$m_1 \simeq m_2 \gg m_3$	$2 \cdot 10^5$	$5 \cdot 10^{10}$	$\gg 2 \cdot 10^{14}$
	$m_1 \simeq m_2 \simeq m_3$	$\gg 4 \cdot 10^3$	$\gg 4 \cdot 10^9$	$\gg 10^{13}$
SMA	$m_1 \ll m_2 \ll m_3$	$\gg 4 \cdot 10^6$	$6 \cdot 10^{10}$	$7 \cdot 10^{15}$
	$m_1 \simeq m_2 \ll m_3$	$2 \cdot 10^5 - 5 \cdot 10^6$	$6 \cdot 10^{10}$	$6 \cdot 10^{14} - 8 \cdot 10^{15}$
	$m_1 \simeq m_2 \gg m_3$	$2 \cdot 10^5$	$6 \cdot 10^{10}$	$\gg 10^{14}$
	$m_1 \simeq m_2 \simeq m_3$	$\gg 4 \cdot 10^3$	$\gg 4 \cdot 10^9$	$\gg 10^{13}$
VO	$m_1 \ll m_2 \ll m_3$	$9 \cdot 10^8$	$5 \cdot 10^{10}$	$\gg 3 \cdot 10^{17}$
	$m_1 \simeq m_2 \ll m_3$	$2 \cdot 10^5 - 5 \cdot 10^8$	$6 \cdot 10^{10}$	$2 \cdot 10^{14} - 5 \cdot 10^{17}$
	$m_1 \simeq m_2 \gg m_3$	$2 \cdot 10^5$	$5 \cdot 10^{10}$	$\gg 2 \cdot 10^{14}$
	$m_1 \simeq m_2 \simeq m_3$	$\gg 4 \cdot 10^3$	$\gg 4 \cdot 10^9$	$\gg 10^{13}$

Only one of the heavy neutrino masses is of the order of magnitude of the scale of the $SU(5) \times U(1)$ symmetry limit. This does not contradict our assumptions: a *democratic* mass matrix with all matrix elements of order $\sim 10^{15} GeV$ would give two vanishing diagonal masses and only one mass of order $10^{15} GeV$.

4 Radiative lepton number violating decays

Of all heavy neutrino decay modes, for example $\nu_L \phi$ or $e^- W^+$, the Higgs channel should dominate. We calculated [8] the decay rate for the process $N \to e^- W^+$ obtaining a lifetime of order of $10^{-8} s$.

Since the heavy neutrinos cannot be produced in accelerators and their lifetime is so low, it will be very difficult to observe them, except, indirectly, through their contribution to radiative processes such as the $\mu \to e\gamma$ decay.

The $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ decays violate lepton number conservation and are not allowed in the standard model. The Majorana mass terms, violate lepton number and these decays could be allowed (figure 1).

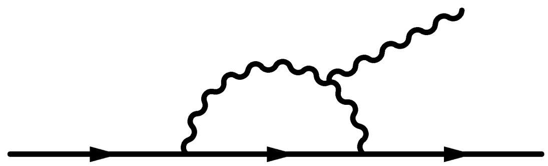

Figure 1: The one-loop diagram for the $\mu \to e\gamma$ decay

The neutrino weak eigenstates are a superposition of three light Majorana neutrinos and three heavy Majorana neutrinos, thus the branching ratios, calculated by Cheng and Li [16], are:

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\overline{\nu})} = \frac{3\alpha}{8\pi} \left| U_{\mu i} U_{ei} \left(\frac{m_i}{M_i} \right)^2 \right|^2 \tag{24}$$

where m_i are the neutrino Dirac masses and M_i are the heavy neutrino masses. For our values of heavy neutrino masses we get: $B \leq 10^{-30}$ for $\mu \to e\gamma$, much smaller than the present limit ($B \leq 10^{-11}$). The branching ratios for the processes $\tau \to \mu\gamma$ and $\tau \to e\gamma$ turn out to be respectively $B \leq 10^{-30}$ and $B \leq 10^{-31}$.

5 Conclusion

After introducing the see-saw mechanism, we have analysed this mechanism assuming the SO(10) model and that the Dirac masses of fermions are generated by a Higgs decuplet. Even if in this way we obtain the bad relationship $M_d = M_l$ we use it to estimate the masses of heavy neutrinos. We have found that not all heavy Majorana neutrino masses are equal and that their mass matrix can not be equal to that of up quarks. We tried to estimate these masses, finding that, in most cases considered, they are: $M_1 = 10^5 - 10^6\ GeV$, $M_2 = 10^9 - 10^{10}\ GeV$ and $M_3 = 10^{13} - 10^{16}\ GeV$. We also calculated that their lifetime is too short to be present in cosmic rays and finally we analysed in which way the heavy neutrino can affect the $\mu \to e\gamma$ decay (and $\tau \to \mu\gamma$, $\tau \to e\gamma$) estimating a rate which is far below the present limit.

References

[1] T.P. Cheng, L.F. Li (1984). *Gauge thoery of elementary particle physics*, Oxford University Press.

[2] E.Kh. Akhmedov, hep-ph/0001311 (2000).

[3] G.B. Gelmini, M. Roncadelli, Phys. Lett. **B99** (1981) 411.

[4] M. Gell-Mann, P. Ramond and R. Slansky in *Supergravity*, ed. by P. van Nieuwenhuizen and D.Z. Freedman (North Holland, Amsterdam, 1979), p.315 T. Yanagida in Proc. of the *Workshop on the Unified Theory and Baryon Number in the Universe*, KEK report 79-18 (1979), p.95 R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44** (1980) 912.

[5] S.M. Bylenky hep-ph/0001311 (2000).

[6] K. Kanaya, Prog. Theor. Phys. **64** (1980) 2278.

[7] See for exemple: S.M. Bilenky, C. Giunti and W.Grimus hep-ph/9812360 (1999).

[8] L.Frassinetti, *Masse dei neutrini: modelli teorici e conseguenze* (Neutrino masses: theoretical models and consequences) Thesis of *Laurea*. Submitted to Bologna University (may 2000).

[9] H. Georgi, (1975) *Particles and fields*, ed. C.E. Carlson.

[10] H. Fritzsch and P. Minkowsky, Ann. Phys. **93** (1975) 193.

[11] P. Langacker, Phys. Rep. **72** (1981) 4.

[12] R.N. Mohapatra and G. Senjanovic, Phis. Rev. **D27** (1983) 1601.

[13] R.N. Mohapatra, (1985) *Unification and Supersymmetry*, ed. Springer-Verlag.

[14] D. Chang, R.N. Mohapatra, J. Gipson, R.E. Marshak and M.K. Parida, Phis. Rev. **D31** (1985) 1718.

[15] M. Abud, F. Buccella, D. Falcone, G. Ricciardi, F. Tramontano, hep-ph/9911238.

[16] T.P. Cheng and L.F. Li Phys. Rev. Lett. **45** (1980) 1908